Magnetic Fields from Heterotic Cosmic Strings

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Large-scale magnetic fields are observed today to be coherent on galactic scales. While there exists an explanation for their amplification and their specific configuration in spiral galaxies – the dynamo mechanism – a satisfying explanation for the original seed fields required is still lacking. Cosmic strings are compelling candidates because of their scaling properties, which would guarantee the coherence on cosmological scales of any resultant magnetic fields at the time of galaxy formation. We present a mechanism for the production of primordial seed magnetic fields from heterotic cosmic strings arising from M theory. More specifically, we make use of heterotic cosmic strings stemming from M5–branes wrapped around four of the compact internal dimensions. These objects are stable on cosmological time scales and carry charged zero modes. Therefore a scaling solution of such defects will generate seed magnetic fields which are coherent on galactic scales today.

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I. INTRODUCTION

We would like to provide a string theoretic explanation of the large-scale magnetic fields observed in the universe today. These galactic magnetic fields, coherent over scales of up to a megaparsec, are observed to be of the order of 10^{-6} G [1]. The dynamo mechanism, whereby turbulence effects serve to amplify seed magnetic fields, can explain both the amplitude and configuration of fields observed in spiral galaxies today, given large enough coherent seed fields. The necessary coherence is nicely explained if these seed fields are generated by string-like objects, which could be produced during phase transitions in the early universe, as was explored in [2].

We attempt to reproduce this mechanism in a string theoretic construction, which would then provide a natural explanation for the existence of large-scale magnetic fields observed in the universe today. The arena to consider is provided by heterotic string theory. Fundamental heterotic strings were ruled out as cosmic string candidates by a stability analysis [3] but heterotic cosmic strings arising from wrapped M5–branes [4] take advantage of a loophole presented in [5] and may provide suitable candidates for cosmic strings that could generate primordial magnetic fields.

In this article we will construct stable heterotic cosmic strings arising from suitably wrapped M5–branes, using [4] as a starting point. In order for these strings to generate galactic magnetic fields, they must both be stable and support charged zero modes. We show that in order for these strings to support such zero modes, a more general picture is required, in which the moduli of a large moduli space of M-theory compactifications are time dependent and evolve cosmologically.

The structure of the paper is as follows. We first give the astrophysical motivation for the problem in Section II, explaining why cosmic strings are relevant to its resolution. Next in Section III we discuss the pion string approach of [2]. In Section IV we present an analogous mechanism in heterotic string theory. We end with a discussion of problems encountered and directions for future research.

II. PRIMORDIAL MAGNETIC FIELDS AND COSMIC STRINGS

A. The dynamo mechanism

It was Fermi who first proposed the existence of a large-scale magnetic field in our galactic disc.[1] He argued that such a field was needed to confine cosmic rays to the galaxy; it would have to have a strength of 10^{-6} to 10^{-5}G. From measurements of synchrotron emission, Faraday rotation, Zeeman splitting and the polarisation of optical starlight, it is now known that the gaseous

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[1] See [6]; the story is related by Parker [7].

disc of the galaxy contains a general azimuthal (toroidal) magnetic field with a strength of 3×10^{-6}G and which is coherent on galactic scales of up to a megaparsec [1, 7, 8]. This field is not only necessary for confinement of cosmic rays, but is responsible for a crucial step in stellar formation and plays an important role in the dynamics of other objects like pulsars and white dwarfs [8].

Moreover, such fields have been detected in many other galaxies, wherever the appropriate measurements have been made, and it is believed that they are ubiquitous in galaxies and galactic clusters. Whereas in spiral galaxies like ours the magnetic field is generally coherent on scales comparable to the visible disc, in elliptical galaxies the coherence length is much smaller than the galactic scale and the fields more random. There has been no detection of purely cosmological fields (fields not associated with any gravitationally bound structure) [9].

Since there are no contemporary sources for galactic fields, they must either be primordial or descended from primordial magnetic fields. These fields would have been present at galaxy formation, and can be reasonably supposed to have condensed along with matter from the original diffuse gas clouds which contracted to form galaxies. However, there are severe observational problems with the hypothesis that these primordial fields are the ones measured today.

Firstly, the gaseous disc of the galaxy rotates non-uniformly, with an angular velocity dependent on the distance r from the axis of rotation. This non-uniform rotation would shear the lines of force of the field into many filaments of alternating signs, contrary to observation. In addition, these fields could not have survived to be observed today. Large-scale magnetic fields in a turbulent medium can escape through various effects which result in a characteristic decay time of 10^8 years, to be contrasted with the galactic lifetime of 10^{10} years [10].

If the original fields could not have survived to present times, we must conclude that the fields we observe are not primordial. In order for fields still to be present at late times despite losses, there must be some process that generates galactic flux continually. This is the turbulent galactic dynamo,[2] which consists of electrically conducting matter moving in a magnetic field in such a way that the induced currents amplify and maintain the original field. We give a brief review of the galactic dynamo in Appendix A.

[2] The classic texts on magneto-hydrodynamics and dynamo theory are [11] and [7], among others. Parker showed in a series of papers [10, 12, 13, 14] that the gaseous disc of the galaxy is a dynamo, and the formal equations on the matter are contained therein and in his 1979 book [7]. There exist many papers on the subject of the galactic magnetic field and its origins (see e.g. [15, 16]).Widrow's review [9] is especially lucid and contains the key references.

B. Seed fields and the coherence length

We have seen that the galactic magnetic fields observed today cannot be primordial and that the dynamo effect provides a mechanism for continual generation of flux. However, seed magnetic fields which are primordial are still required.[3] This can be seen by considering the hydro-magnetic equation (A.1), which is linear and homogeneous in $\vec{B}$ and contains no source term. Seed fields must therefore have been present to be amplified by the dynamo mechanism. To determine the strength of the seed field required in order to obtain magnetic fields of order 10^{-6}G today, two effects must be considered. Firstly, magnetic fields will be amplified during galaxy formation by the stretching and compression of field lines that occur during the collapse of gas clouds to form galaxies. In spiral galaxies these processes can amplify a primordial field by several orders of magnitude [9]. Amplification after galaxy formation is via the dynamo mechanism and is given by Γ, the growth rate for the dominant mode of the dynamo. The amplification factor $\mathcal{A}$ by which the magnetic field grows between times t_i and t_f after galaxy formation is then

$$\mathcal{A} = \frac{B_f}{B_i} = e^{\Gamma(t_f - t_i)}. \tag{2.1}$$

The maximum amplification factor is given in [9] as $\mathcal{A} = 10^{14}$, implying that a seed field with strength of at least 10^{-20}G is required. However, it must be noted that this minimum could increase. Observations of microgauss fields in galaxies at a redshift of 2 shorten the time available for dynamo action and lead to a seed field as large as 10^{-10}G [9]. Similarly, imperfect escape of field lines may allow only a limited amplification of the mean field [17].

Various mechanisms for generating the seed magnetic fields have been suggested, but coherence over the lengths required is not easily explained unless one makes use of a scaling string network. The challenge is the following: the seed magnetic fields need to be coherent on cosmological scales. More specifically, the comoving distance corresponding to the mean separation of galaxies has a physical size $\lambda_{\rm gal}$ similar to the Hubble radius $H(t)^{-1}$ at the time t_{eq} of equal matter and radiation, the time when structures on galactic scales can start to grow by gravitational instability. This is a very late time from a particle physics perspective (see Figure 1).

Typical particle physics processes will create magnetic fields whose coherence length is limited by the Hubble radius at the time t_{pp} when the processes take place (i.e. in the very early universe). In fact, the coherence scale is typically microscopic even at that time. Even if the

[3] In fact, large-scale dynamo action in a galaxy is preceded by a small-scale dynamo that prepares the seed fields for the former [1].

coherence scale expands with the cosmological expansion of space, it will be many orders of magnitude smaller than the Hubble radius at t_{eq} since the coherence length scales with $t^{1/2}$ whereas the Hubble radius is growing linearly in t.

Thus, explaining the coherence of the seed magnetic fields at the time corresponding to the onset of galaxy formation is a major challenge for attempts to generate seed magnetic fields using ideas from particle physics. A particle physics source that will scale appropriately so as to avoid this problem is given by cosmic strings.

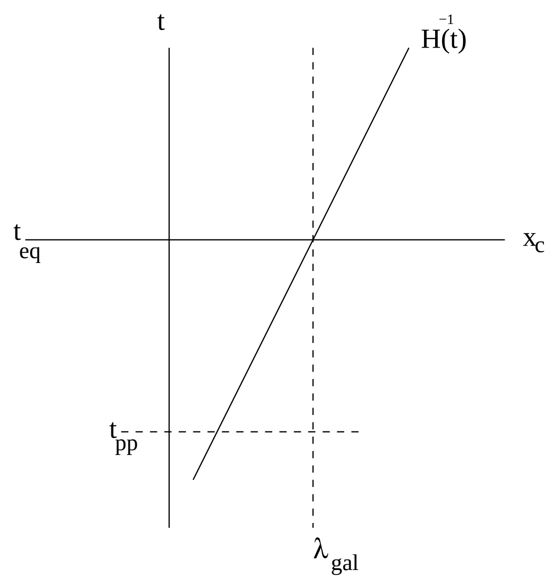

FIG. 1: The coherence problem

C. Cosmic strings

With the physics of the early universe described by a theory which undergoes spontaneous symmetry breaking, the universe is expected to have gone through various phase transitions as it cooled. Such transitions can rather generically lead to the formation of topological defects (see e.g. [18, 19, 20] for reviews on topological defects in cosmology): configurations of energy which are topologically stable, where the topology in question is that of the vacuum manifold. Upon cooling past some critical temperature T_c (corresponding to the time t_c) the Higgs field at a point $\mathbf{x}$ in space acquires a vacuum expectation value $\langle\phi(\mathbf{x})\rangle$, corresponding to some point in the vacuum manifold $\mathcal{M}$.

If $\mathcal{M}$ consists of more than a single point, $\langle\phi(\mathbf{x})\rangle$ will be chosen randomly for points in space separated by more than some correlation length ξ, where ξ is bounded above by the Hubble radius at time t_c (by causality) but is typically much smaller. Specifically, if matter is in thermal equilibrium before the transition, then the initial correlation length at the time of the phase transition is a microphysical scale, the so-called *Ginsburg length* [21]. Depending on the topology of the vacuum manifold, this random distribution of field values in the vacuum manifold will lead to the formation of topological defects of different dimensions. It is when an axial or cylindrical symmetry is broken that a linelike defect or string forms, because the vacuum manifold is not simply connected. In other words, if $\mathcal{M}$ has a non-trivial first homotopy group $\Pi_1(\mathcal{M}) \neq 1$, the defects are cosmic strings, which can be macroscopic. Thus cosmic strings are not fundamental strings but topological defects formed during phase transitions as the universe cooled.

During a phase transition, a network of cosmic strings will form with a characteristic length scale comparable to ξ. This correlation length gives both the typical curvature radius of the strings as well as the typical distance between neighbouring strings. As the universe expands, so will the correlation length $\xi(t)$. The string network can be separated into the so-called infinite strings (strings with curvature radius larger than the horizon at the time t) and a distribution of string loops with radii R smaller than t which are formed when the infinite strings intercommute (the strings cannot have free ends). The loops will oscillate and emit gravitational radiation. This way, sufficiently small loops will radiate all their energy away and decay. This means that an initially dense string network will be diluted as the strings chop each other up and the resulting loops decay. This is also the scenario favoured by entropy considerations.

However, by causality the correlation length ξ can never grow larger than t, since this would imply the presence of correlations in the position of the field in the vacuum manifold over lengths greater than the distance light could have travelled. The rate at which the strings can chop each other off into loops is thus limited by the speed of light. This means that either the network approaches a scaling solution in which ξ remains a fixed fraction of t or it grows more slowly, in which case $t/\xi(t)$ increases. In the latter case strings would eventually come to dominate the total energy density of the universe. Using Boltzmann-type equations [20] describing the energy transfer between the network of infinite strings and the distribution of loops it can be shown analytically that for non-superconducting strings and for time-independent string interaction cross-sections $\xi(t)/t$ is bounded from below and thus this latter case cannot occur.

It has been verified using sophisticated numerical string network evolution simulations [22, 23, 24] that instead the distribution of infinite strings will converge to a *scaling solution* in which $\xi(t)/t$ is independent of time and the string density is constant relative to the rest of the radiation and matter energy density in the universe. It is called a scaling solution because, scaled to the Hubble radius, the string network looks the same at all times. The string properties, such as $\xi(t)$, are proportional to the time passed [18, 19, 20].

If a mechanism for production of primordial magnetic

fields by cosmic strings can be found, the scaling of the string network will provide a natural explanation for the coherence of the resulting magnetic fields over cosmological scales at late times, giving rise to seed fields which are coherent on galactic scales at the time of galaxy formation.

III. PION STRINGS

Exactly such a mechanism was proposed in [2], for the case of pion strings. These arise as global vortex line solutions of the effective QCD Lagrangian below the chiral symmetry breaking scale of $T_c \sim 100$ - 200 MeV, as shown in [25]. These pion strings couple to electromagnetism via anomalous Wess-Zumino–type interactions. Using the results of Kaplan and Manohar [26] for such a coupling, it can then be shown that pion strings could generate seed magnetic fields greater than 10^{-20}G and coherent on comoving scales of a few kiloparsec, as required, provided the strings reach scaling soon enough.[4]

A. Anomalous coupling to electromagnetism: the Kaplan-Manohar mechanism

In [26] the authors considered a theory with a single Dirac fermion ψ coupled to a complex neutral scalar field ϕ. The Lagrangian is

$$\begin{aligned}\mathcal{L} \;&=\; \imath\bar{\psi}\not{D}\psi + |\partial_\mu\phi|^2 - g\phi\bar{\psi}_L\psi_R - g\phi^*\bar{\psi}_R\psi_L \\ &\quad -\frac{\lambda}{2}\left(|\phi|^2 - f^2\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (3.2)\end{aligned}$$

and the theory has a local U(1) electromagnetic symmetry (where ψ has charge 1 and ϕ is neutral) and a global $\mathrm{U}_A(1)$ symmetry under which

$$\begin{aligned}\psi_L \;&\rightarrow\; e^{\imath\alpha}\psi_L, \\ \psi_R \;&\rightarrow\; e^{-\imath\alpha}\psi_R, \\ \phi \;&\rightarrow\; e^{2\imath\alpha}\phi.\end{aligned}$$

The symmetry is spontaneously broken by the vacuum expectation value $\langle\phi\rangle = f$. Then we are left with a massive scalar with mass $m_s = \sqrt{\lambda}f$, a massive fermion with $m_e = gf$, a massless photon and a massless pseudoscalar Goldstone boson, termed the axion a.

Because of the $U_A(1)$ anomaly, the axion couples to photons via the Adler-Bell-Jackiw triangle diagram. At low enough energies, only the massless particles are important, as in the low-energy effective Lagrangian obtained by integrating out the heavy particles:

$$\mathcal{L} \;=\; \frac{1}{2}(\partial_\mu a)^2 - F\wedge\star F - \frac{e^2}{32\pi^2}\left(\frac{a}{f}\right)F\wedge F\,. \qquad (3.3)$$

Then the equation of motion for the electromagnetic field is

$$dF \;=\; -\frac{\alpha}{\pi}d\left(\frac{a}{f}\right)\star F\,, \qquad (3.4)$$

which makes manifest the coupling between the axion and the photons.

The model given by (3.2) has vacuum manifold $\mathcal{M} = S^1$, which has first homotopy group $\Pi_1(\mathcal{M}) = \mathbb{Z}$ and hence admits vortex (cosmic string) solutions given by

$$\phi(r,\theta) \;=\; f(r)e^{\imath\theta}\,, \qquad (3.5)$$

where $f(r) \rightarrow 0$ as $r \rightarrow 0$, and $f(r) \rightarrow f$ as $r \rightarrow \infty$. In the above, r and θ are the polar coordinates in the plane perpendicular to the vortex, and $r = 0$ corresponds to the center of the vortex. The vortex solution (3.5) corresponds to the axion varying as we rotate about the vortex. Thus, via (3.4), the vortex is coupled to the photons. Specifically, if the vortex carries a current, then the axionic coupling leads to a magnetic field circling the string.

To find the electromagnetic fields arising from this vortex configuration with current flowing along the vortex, we solve Maxwell's equations (3.4) in the presence of the vortex string. This is accomplished by taking $\frac{a}{f} = \theta$ in (3.4). One finds two static z-independent solutions [26]:

$$\begin{aligned}E_r \;&=\; c_+r^{-1-\frac{\alpha}{\pi}} + c_-r^{-1+\frac{\alpha}{\pi}}, \qquad (3.6)\\ B_\theta \;&=\; c_+r^{-1-\frac{\alpha}{\pi}} - c_-r^{-1+\frac{\alpha}{\pi}}.\end{aligned}$$

Since $B_\theta = \pm E_r$, the solutions have the Lorentz transformation properties expected if the vortex were to carry a light-like current 4-vector $j^\mu = (\lambda, 0, 0, \pm\lambda)$. This indicates that the charge carriers move along the vortex at the speed of light. From the fermionic zero modes, found by solving the Dirac equation for ψ in the vortex background, Kaplan and Manohar were able to solve for $c_\pm$, finding

$$E_r \;=\; -B_\theta \;\sim\; r^{-1+\frac{\alpha}{\pi}}, \qquad (3.7)$$

so the fall-off is slower than expected classically ($\frac{1}{r}$). The decay rate depends on the strength of the anomalous coupling. In the model which we present below, we have $\alpha = 0$.

[4] The interaction of cosmic strings with magnetic fields has been discussed in many papers, starting with [27], but their possible connection to the primordial seed fields needed to explain current observations of galactic fields (both being produced during phase transitions in the early universe) was first suggested in [28] and then elaborated on in [29]. The importance of the coherence length was not commented on until fairly recently [2]. Note that a different mechanism of magneto-genesis from cosmic strings was proposed in [30], in which it was argued that vortices in the plasma could be formed by cosmic string loops, and that these vortices could produce magnetic fields by the Harrison-Rees effect. The approach here is rather to show that the strings produce the seed magnetic fields directly.

B. Pion strings

The lagrangian (3.2) was generalised in [2] to the case of the low energy nonlinear σ model for QCD with two species of massless quarks. The model has two complex scalar fields, the first containing the charged pions $\pi^{\pm}$, the second the neutral pion π^0 and the σ field. It is convenient to write the fields in an $SU(2)$ basis as

$$\Phi = \sigma\frac{\sigma^0}{2} + \imath\vec{\pi}\cdot\frac{\vec{\tau}}{2}, \qquad (3.8)$$

where σ^0 is the unit matrix and the τ_i are the Pauli matrices. The bosonic part of the Lagrangian is

$$\mathcal{L}_\Phi = \mathrm{tr}\left[(\partial_\mu\Phi)^+\partial^\mu\Phi\right] - \frac{\lambda}{2}\left[\mathrm{tr}(\Phi^+\Phi) - f^2\right]^2. \qquad (3.9)$$

In addition the Lagrangian will contain the standard kinetic terms for the left- and right-handed fermion SU(2) doublets Ψ_L and Ψ_R. The Yukawa coupling term takes the form

$$\mathcal{L}_I = g\bar{\Psi}_L\Phi\Psi_R + h.c., \qquad (3.10)$$

where h.c. stands for Hermitean conjugate.

After spontaneous symmetry breaking, there are 3 Goldstone bosons, the massless pions $\vec{\pi}$, and a massive σ particle:

$$\phi = \frac{\sigma + \imath\pi^0}{\sqrt{2}};\ \pi^{\pm} = \frac{\pi^1 \pm \imath\pi^2}{\sqrt{2}}. \qquad (3.11)$$

As shown in [25], this model admits vortex solutions, but not of the stable type since the vacuum manifold is $\mathcal{M} = S^3$ and hence has trivial first homotopy group. The vortex solutions of this model are of the *embedded* type. They are obtained by setting $\pi^{\pm} = 0$ and considering the vortex solution (3.5) of the reduced two-field system where only ϕ is allowed to be non-vanishing. The resulting vortex solution is called the *pion string*. Pion strings are unstable in the vacuum since the winding of ϕ can disappear by $\pi^{\pm}$ being excited. However, as was argued in [31], electromagnetic plasma effects in the early universe will create an effective potential which drives $\pi^{\pm}$ to zero while not affecting ϕ (to leading order). Thus, one can apply the usual topological and dynamical arguments for defect formation to the pion string model and conclude that after the QCD phase transition a network of pion strings will form which will be stabilized by the electromagnetic plasma until the time of recombination.

C. Pion strings and the Kaplan-Manohar mechanism

In order to be able to apply the KM mechanism to argue for generation of magnetic fields by cosmic strings, one also requires the existence of current on the strings. Such current will automatically be generated at the time of the phase transition (along with defect formation) provided that the strings admit charged zero modes, i.e. provided that the strings are superconducting [32]. The pion strings are superconducting [2], and hence magnetic fields coherent with the strings are automatically generated during the phase transition, as set out in [2].

The key point is that the cosmic string network continues to generate magnetic fields for all times. The network of magnetic field lines stretches as the string network stretches. Hence, the correlation length of the magnetic fields generated by the string network scales as $\xi(t)$. Pion strings eventually decay (at a time which we denote by t_d). The final correlation length of the magnetic fields set up by these strings will be given by the comoving distance corresponding to $\xi(t_d)$, which is of the order of the Hubble radius at that time. Provided that pion strings decay later than the time corresponding to a temperature of 1 MeV, this final correlation length will be of the size of a galaxy. Note that in this model, there is an upper cut-off on the scale of coherent magnetic fields. Magnetic fields on supergalactic scales can arise only as a random superposition of galactic scale fields, and hence the power spectrum of magnetic fields will be Poisson-suppressed on these scales.

In Section IV, we will discuss a model for generating primordial magnetic fields along the lines of [2] starting from superstring theory. The strings which we will make use of will turn out to be stable. Hence, unlike in the pion string model, there will be no upper cutoff on the scale of coherent magnetic fields. Coherent magnetic fields on the scale of galaxy clusters and above will result. However, the dynamo will obviously only amplify the galactic magnetic fields and not the magnetic fields on larger scales.

D. Cosmic strings and magnetic fields

Cosmic strings carry energy and hence can become seeds for gravitational instability. According to an early scenario of structure formation triggered by cosmic strings [33, 34, 35], there was a one-to-one correspondence between string loops and cosmological objects. At each time t, loops with radius $R \sim t$ are produced by the interactions of the infinite string network. The currents on the infinite strings induce currents on the loops, and these loop currents will induce magnetic fields about the loops whose coherence length remains constant in comoving coordinates (and does not grow as $\xi(t)$). Thus, in this scenario the galactic magnetic field is a remnant of the magnetic field of the string loop which seeded the galaxy.

This cosmic string scenario relies on there being more energy in the distribution of string loops than in the long string network. According to more recent cosmic string simulations, this might not be the case. If the infinite string network dominates, then most of the structure formation triggered by strings occurs in the wake-like over-

densities [36, 37] behind long moving strings. Galaxy formation will occur in these wakes, and thus the galaxies will inherit the magnetic fields present in them. The coherence scale of these fields is then comparable to or larger than the size of the regions which collapse to form galaxies. Thus galaxies will also inherit coherent magnetic fields in this scenario.

Recent cosmic microwave anisotropy data constrains the contribution of cosmic strings to the power of density inhomogeneities in the universe to at most 10% [38]. In a model with cosmic strings contributing at a level not too far below this bound, the strings will help trigger galaxy formation in their wakes. In fact, for a smaller value of the string tension, decay of string loops by gravitational radiation is slower. Thus, more cosmic string loops will be present in over-dense regions, and they will transfer their coherent magnetic fields to the resulting galaxies. The bottom line is that even in cosmic string models in which strings contribute to structure formation at a level consistent with the current bounds, coherent magnetic fields on galactic scales are induced.

IV. HETEROTIC COSMIC STRINGS

It is our aim to reproduce the results of [2] for cosmic strings arising in a string theoretic setting. We would then have an explanation for seed magnetic fields with the required coherence scale that was consistent with string theory as the theory of the early universe. We begin by considering heterotic cosmic strings. We require our cosmic strings to be stable, and that they carry charged zero modes. We consider heterotic strings because charge is evenly distributed over them instead of being localised at the end-points. The gauge group (either SO(32) or $E_8 \times E_8$) comes from charged modes that propagate only on the string. In addition, compactifications of the heterotci string have led to the most phenomenologically attractive vacua in the string/M-theory landscape. Vacua containing exactly the MSSM spectrum from heterotic compactifications were constructed in [41], and other realistic vacua have been constructed (see [42, 43, 44] for instance). However, as we shall see in Section IV A, fundamental heterotic strings cannot give rise to stable cosmic strings upon dimensional reduction. Instead we have to use wrapped M-branes in a higher dimensional theory. Their stability is discussed in Section IV B and the existence of charged zero modes on the suitable candidates is discussed in Section IV C.

A. The axionic instability

Unfortunately, fundamental heterotic strings were ruled out as candidates for cosmic strings by Witten in 1985 [3]. Although simple decay is ruled out because there are no open strings in the theory,[5] Witten argues that the fundamental heterotic string is actually an axionic string, and as a result is unstable. The argument runs as follows: first, the worldsheet theory is anomalous because current is carried in one direction only. Then anomaly cancellation generically demands the presence of axions. During phase transitions as the universe cools axionic domain walls are formed, the boundaries of which must be superconducting. The heterotic strings become the boundaries of an axionic domain walls. The tension of the domain wall leads to an instability of the string towards its contraction. The instability can be seen by considering the energy of a large circular string [3]

$$E = \frac{R}{\alpha\prime} + \pi R^2 \sigma \,, \tag{4.12}$$

where R is the radius of the string, σ is the wall tension, and the second term thus represents the energy due to the domain wall tension (the tension σ being the energy per unit area of the domain wall). This term dominates when $R > \frac{1}{\alpha'\sigma}$, and in this regime the string therefore tends to collapse. As the domain wall shrinks, strings intersect and chop each other off, until $R < \frac{1}{\alpha'\sigma}$. Then the string mass alone determines the energy of the string. Microscopic strings will decay away quickly through gravitational radiation. The string is prevented from growing to the cosmic scales at which it could survive by the domain wall [40].

Fundamental heterotic strings were also ruled out by Witten [3] as viable cosmic string candidates on tension grounds. In perturbative string theory about a flat background, the string tension is too large to be compatible with the existing limits [38].

B. Loopholes via M-theory and the BBK construction

The possibility of obtaining stable cosmic superstrings was resurrected by Copeland, Myers and Polchinski [5] (see also [46] and the review in [47]). The existence of extended objects of higher dimension, namely branes of various types, provides a way to overcome the instability problems pointed out by Witten [3], as we shall see for the heterotic string in particular. On the other hand, string tensions can in general be lowered by placing the strings in warped throats of the internal manifold and using the gravitational redshift to reduce the string tensions, so that this constraint no longer rules out all cosmic superstrings.

Using the axionic instability loophole presented in [5], Becker, Becker and Krause [4] studied the possibility of

[5] Note that this is not necessarily the case for the $SO(32)$ heterotic string which can end on monopoles. This was pointed out by Polchinski [45].

cosmic strings in heterotic theory, pointing out that suitable string candidates can arise from wrapped branes in M theory. When compactified on a line segment $S^1/\mathbb{Z}_2$, M theory reduces to heterotic string theory [48]. Compactifying a suitable configuration to $3+1$ dimensions could give us heterotic cosmic strings in our world. Note that because brane tensions are significantly lower than the fundamental string tension, the cosmic strings arising from such wrapped branes can also avoid the tension bound mentioned above.

There are two kinds of M-theory branes to consider as potential cosmic string candidates: M2– and M5–branes. In descending to $3+1$ dimensions, suitable candidates must be extended along the time direction and one of the large spatial dimensions. They must therefore wrap 1- or 4-cycles respectively in the internal dimensions. Heterotic string theory is obtained by compactifying M theory on $S^1/\mathbb{Z}_2$, so the internal dimensions are naturally separated into x^{11} along the circle, and $x^4, .., x^9 \in CY_3$ on the 10-dimensional boundaries of the space, which we can think of as M9–branes. Thus there are 4 possible wrapped-brane configurations, which can be labelled (following the notation of [4]) as $\mathrm{M2}_\perp$, $\mathrm{M2}_\parallel$, $\mathrm{M5}_\perp$ and $\mathrm{M5}_\parallel$, where the designations perpendicular and parallel refer to the brane wrapping and not wrapping the orbifold direction x^{11} respectively. Their viability as cosmic string candidates is discussed below.

Wrapped M2–branes

There is no 1–cycle available in a Calabi-Yau threefold, so the M2–brane candidates can only wrap x^{11}. We can check their viability by comparing the tension of the resulting cosmic strings with the constraint given by anisotropy measurements of the CMB: [6]

$$\mu G_N \ \leq \ 2 \times 10^{-7}, \tag{4.13}$$

where G_N is Newton's gravitational constant.

The M2–brane action is given by

$$S_{M2} \ = \ \tau_{M2} \int dt \int dx \int_0^L dx^{11} \sqrt{-\det h_{ab}} + \dots\,, \tag{4.14}$$

where τ_{M2} is the tension of the brane, and h_{ab} denotes the worldsheet metric. The 11 dimensional metric G_{IJ} of spacetime is found by considering the internal manifold to be compactified by the presence of G-fluxes [51]. The result is

$$\begin{aligned} ds^2_{11} \ &= \ e^{-f(x^{11})} g_{\mu\nu} dx^\mu dx^\nu \\ &\quad + e^{f(x^{11})} \left(g_{mn} dy^m dy^n + dx^{11} dx^{11} \right)\,, \end{aligned} \tag{4.15}$$

where

$$e^{f(x^{11})} \ = \ (1 - x^{11} Q_v)^{2/3}\,. \tag{4.16}$$

In the above $g_{\mu\nu}$ is the metric in our four dimensional spacetime, and g_{mn} is the metric on the Calabi-Yau threefold. There is warping along the orbifold direction given by the function $f(x^{11})$, and Q_v is the twobrane charge. Making use of the above metric, we obtain from (4.14) the following cosmic string action:

$$\begin{aligned} S_{M2} \ &= \ \mu_{M2} \int dt \int dx \sqrt{-g_{tt} g_{xx}} + \dots, \qquad (4.17) \\ \mu_{M2} \ &= \ \tau_{M2} \int_0^L dx^{11} e^{-f(x^{11})/2}, \\ &= \ \frac{3\tau_{M2}}{2Q_v} \left[1 - (1 - LQ_v)^{2/3} \right]. \end{aligned}$$

Upon evaluation, this gives a brane tension of

$$\mu_{M2} \ \approx \ 9(2^{10}\pi^2)^{1/3} M^2_{GUT}, \tag{4.18}$$

which is too large to satisfy the bound (4.13). Thus wrapped M2–branes are ruled out as candidates for heterotic cosmic strings. However, they are stable (see [4]). If produced in a cosmological context, they would therefore have disastrous consequences.

Wrapped M5–branes: Tension

For the case of the M5–brane, there are two possible types of configurations. Following [4] we label them $\mathrm{M5}_{||}$ and $\mathrm{M5}_\perp$. The $\mathrm{M5}_{||}$–brane is confined to the 10-dimensional boundary of the space, wrapping a 4-cycle Σ_4, while the $\mathrm{M5}_\perp$–brane wraps x^{11} and a 3-cycle Σ_3. By similar analyses to those outlined above one obtains the brane action for the parallel five-brane:

$$S_{M5_{||}} \ = \ \tau_{M5} \int dt dx \int_{\Sigma_4} d^4 y \sqrt{-\det h_{ab}} + \dots\,, \tag{4.19}$$

where τ_{M5} is the brane tension. The effective string tension from the point of view of four-dimensional spacetime is given by

$$\mu_{M5_{||}} \ = 64 \left(\frac{\pi}{2} \right)^{1/3} \left(1 - \frac{x^{11}}{L_c} \right)^{2/3} M^2_{GUT} r^4_{\Sigma_4}, \tag{4.20}$$

where r_{Σ_4} measures the mean radius of the 4-cycle Σ_4 in units of the inverse GUT scale. L_c is a critical length of the $S^1/\mathbb{Z}_2$ interval determined by G_N.[7]

[6] This limit is given in [49] and [38] where WMAP and SDSS data was used. A tighter bound of 10^{-8} is suggested by analysis of limits on gravitational waves from pulsar timing observations [50]. However, these pulsar bounds are not robust since they depend sensitively on the distribution of cosmic string loops which is known rather poorly.

[7] See [51] and [52] for the derivations.

Similarly, for the orthogonal five-brane one obtains

$$S_{M5_\perp} = \tau_{M5} \int dt dx \int_0^L dx^{11} \int_{\Sigma_3} d^3y \sqrt{-\det h_{ab}} + \ldots.. , \quad (4.21)$$

and the associated cosmic superstring tension is

$$\mu_{M5_\perp} = \frac{1152}{5} \frac{\pi}{2}^{1/3} M_{GUT}^2 r_{\Sigma_3}^3 , \quad (4.22)$$

where r_{Σ_3} measures the mean radius of the 3-cycle Σ_3 in units of the inverse GUT scale. Although there is some dependence on the size of the wrapped space, it is not hard for the $M5_{||}$–brane to pass the CMB constraint. With a little more difficulty, the $M5_\perp$ brane also passes this test (although the numerical coefficient given in (3.23) of [4] is about an order of magnitude too small).

Wrapped M5–branes: stability

The next check is a stability analysis, which shows that only the $M5_{||}$–brane is stable. The reason is that axionic branes are unstable [3]. The massless axion that is responsible for this instability can only be avoided in the case of the M5–brane on the boundary: $M5_{||}$. The argument is presented in detail in [4] and is sketched below (see also [5, 46]).

To begin with, the presence of a massless axion is generally implied by the existence of the branes. M5–branes are charged under C_6 (the Hodge dual to C_3 in 11 dimensions). This form descends to C_2 in the 4-dimensional theory and, via

$$\star\, dC_2 = d\phi, \quad (4.23)$$

this implies the presence of an axionic field. However, the presence of the M9 boundaries leads to a modification of $G = dC_3$ on the boundaries. Together with appropriate U(1) gauge fields, this leads to a coupling of C_2 to the gauge fields. This amounts to a Higgsing of the gauge field which then acquires a mass given by the axion term.

To see how this happens, recall that because of the presence of the boundaries on which a 10-dimensional theory lives, an anomaly cancellation condition must be satisfied. Writing the 10-dimensional anomaly as $I_{12} = I_4 I_8$ we require for anomaly cancellation the existence of a two-form B_2 such that $H = dB_2$ satisfies

$$dH = I_4 . \quad (4.24)$$

In addition, it is required that the interaction term

$$\Delta L = \int B_2 \wedge I_8 \quad (4.25)$$

be present [48]. In M theory the four-form I_4 is promoted to a five-form I_5, and although $dG = 0$ (a Bianchi identity) in the absence of boundaries, we must have

$$dG \sim \delta(x^{11}) dx^{11} I_4 \quad (4.26)$$

in the presence of boundaries. Thus, the Bianchi identity acquires a correction term which turns out to be [48]

$$dG = c\kappa^{\frac{2}{3}} \delta\left(\frac{x^{11}}{L}\right) \left(d\omega_Y - \frac{1}{2} d\omega_L\right) , \quad (4.27)$$

written in terms of the Yang-Mills three-form ω_Y and the Lorentz Chern-Simons three-form ω_L given by

$$\begin{aligned} d\omega_Y &= \text{tr } F \wedge F; \\ d\omega_L &= \text{tr } R \wedge R. \end{aligned} \quad (4.28)$$

Then

$$G = dC_3 + \frac{c}{2}\kappa^{\frac{2}{3}} \left(\omega_Y - \frac{1}{2}\omega_L\right) \epsilon(x^{11}) \wedge dx^{11}$$

which implies

$$H = dB_2 - \frac{c}{2L}k^{\frac{2}{3}} \left(\omega_Y - \frac{1}{2}\omega_L\right) . \quad (4.29)$$

It follows that $H \wedge \star H$ contains the term

$$\left(\omega_Y - \frac{1}{2}\omega_L\right) \wedge dC_6 \quad (4.30)$$

which upon integration (and integrating by parts) yields

$$\int C_6 \wedge \left(\text{tr } F \wedge F - \frac{1}{2}\text{tr } R \wedge R\right) . \quad (4.31)$$

Note that C_6 is in the M5–brane directions here.

From earlier work we know the gauge group is generically broken to something containing a U(1) factor, so there exists some F_2 on the boundary. Then the 11D action is

$$S_{11D} = -\frac{1}{2 \times 7! \kappa_{11}^2} \int_{\mathcal{M}^{11}} |dC_6|^2 + \frac{c}{2\kappa_{11}^{\frac{4}{3}}} \int_{\mathcal{M}^{10}} C_6 \wedge \text{tr } F \wedge F - \frac{1}{4g_{10}^2} \int_{\mathcal{M}^{10}} |F|^2 \quad (4.32)$$

which dimensionally reduces to

$$S_{4D} = -\frac{1}{2} \int_{\mathcal{M}^4} |dC_2|^2 + m \int_{\mathcal{M}^4} C_2 \wedge F_2 - \frac{1}{2} \int_{\mathcal{M}^4} |F_2|^2 \quad (4.33)$$

where

$$m \propto \frac{L_{\text{top}}^4}{V^{\frac{1}{2}} V_h^{\frac{1}{2}}} , \quad (4.34)$$

V being the CY volume averaged over the $\frac{S^1}{\mathbb{Z}_2}$ interval and V_h the CY volume at the boundary. L_{top} is a length parameter defined by

$$\int_{\mathcal{M}^{10}} C_6 \wedge tr(F \wedge F_2) = L_{\text{top}}^4 \int_{\mathcal{M}^4} C_2 \wedge F_2 .$$

The equations of motion for A_1 and C_2 are found to be

$$d\star_4 dA_1 = -mdC_2; \quad (4.35)$$
$$d\star_4 dC_2 = -mF_2. \quad (4.36)$$

(4.36) is solved by taking $dC_2 = \star(d\phi - mA_1)$ which gives

$$d\star dA_1 = \star(-md\phi + m^2 A_1). \quad (4.37)$$

For the ground state in which $\phi = 0$ or by picking a gauge which sets $d\phi = 0$, this result shows that A_1 has acquired a mass m.

$$A_1 \to A_1 - \frac{d\phi}{m}. \quad (4.38)$$

The U(1) gauge field has swallowed the axion ϕ and become massive. The theory no longer contains an axion.

In order for this anomaly cancellation mechanism (which swallows the axion and thus eliminates the instability of the strings) to work, the gauge field must be on the boundary and thus the brane must be parallel to the boundary. Thus, only the $M5_{||}$–brane is stabilized, and the $M5_{\perp}$–brane remains unstable.

C. Charged zero modes on the strings

We now need to argue for the existence of charged zero modes (we will focus on fermionic zero modes) on the strings arising from wrapped $M5_{||}$–branes. In $1+1$ dimensions, the degrees of freedom of free fermions and free bosons match, and the corresponding conformal field theories can be shown to be equivalent. This is not the case in higher dimensions, where spin degrees of freedom distinguish between them. This observation is at the heart of bosonisation, the process of going from a fermionic basis to a bosonic basis. In evaluating the superconductors on the string resulting from the wrapped M5–brane, we find that the correct basis is a charged fermionic one, implying fermionic superconductivity.

Here we derive the coupling to electromagnetism that can arise on the worldsheet of the heterotic cosmic string and argue using inverse bosonisation (fermionisation) that this can be recast in a more familiar form by writing it in terms of fermions. What results is an explicit kinetic term for charged fermions on the worldsheet.[8]

Coupling to Electromagnetism

Consider a wrapped $M5_{\|}$–brane. It can be taken to be along the following directions:

$$M5_{\|} \quad 0\ 1\ 4\ 5\ 6\ 7$$

[8] We would like to thank Ori Ganor for directing our attention to the applicability of bosonisation in our case.

Let the $0,1$ co-ordinates be labelled by x and the remaining co-ordinates wrapped on Σ_4 be labelled by y. The massless field content on the five-brane worldvolume is given by the tensor multiplet $(5\phi, B^+_{mn})$ [61, 62, 63], where the scalars correspond to excitations in the transverse directions and the tensor is antisymmetric and has antiself-dual field strength $H_3 = dB^+$. Thus it has $3 = \frac{1}{2} \times {}^4C_2$ degrees of freedom which, together with the scalars, make up the required 8 bosonic degrees of freedom.[9]

The field strength H_3 couples to C_3, the bulk three-form field sourced electrically by the M2–brane and magnetically by the M5–brane, as given in [64]:

$$S = -\frac{1}{2}\int d^6\sigma\sqrt{-h}[h^{ij}\partial_i X^M \partial_j X^N g_{MN} \quad (4.39)$$
$$+\frac{1}{2}h^{ij}h^{jm}h^{kn}(H_{ijk} - C_{ijk})(H_{lmn} - C_{lmn}) - 4],$$

which can be rewritten in terms of differential forms as

$$S = -\frac{1}{2}\int d^6\sigma\sqrt{-h}\left(h^{ij}g_{ij} - 4\right) \quad (4.40)$$
$$-\frac{3}{2}\int (H_3 - C_3)\wedge\star(H_3 - C_3).$$

Here $i,j = 0,1,...,5$ are indices on the brane worldvolume and $M,N = 0,...,9,11$ are indices in the full eleven-dimensional theory. g_{ij} is the pullback of the 11-dimensional metric, C_{ijk} is the pullback of the 11-dimensional three-form, and h is the auxiliary worldvolume metric. Explicitly,

$$g_{ij} = \partial_i X^M \partial_j X^N g^{(11)}_{MN}; \quad (4.41)$$
$$C_{ijk} = \partial_i X^M \partial_j X^N \partial_k X^P C^{(11)}_{MNP}. \quad (4.42)$$

B^+ and C_3 are both functions of y as well as x. To find the massless modes on the string upon compactification on X, we decompose them in terms of harmonic forms. For a harmonic differential form β on a closed compact manifold (such as Σ_4) we have $d\beta = d\star\beta = 0$. The two-form is decomposed as

$$B^+ = \phi^a(x)\otimes\Omega^a_2(y) + b_2(x)\otimes\Phi(y); \quad (4.43)$$
$$dB^+ = d\phi^a(x)\otimes\Omega^a_2(y), \quad (4.44)$$

where a runs over the two-cycles on the Σ_4 which the M5-brane wraps,[10] We have taken $H^1(\Sigma_4) = 0$ for simplicity.

[9] A $D = 11$ Majorana spinor has 32 real components, which are reduced to 16 by the presence of the M5–brane. This means the M5–brane theory will have 16 fermionic zero modes and 8 bosonic zero modes [62].

[10] We take Ω^a_2 to be antiselfdual, so that $a = 1, ..., b_-$, where we have chosen a basis of $H^2(\Sigma_4)$ made of (b_+) forms which are entirely selfdual and (b_-) forms which are entirely antiselfdual. This imposes the property of antiselfduality mentioned earlier for the two-form living on the five-brane. (Clearly then, $\mathrm{Dim}\, H^2(\Sigma_4) = b_- + b_+$.)

Ω_2^a are the harmonic two-forms on Σ_4 and b_2 is a two-form in the $0,1$ directions. Similarly we want C_3 to be decomposable as

$$C_3 \;=\; A^a(x) \otimes \tilde{\Omega}_2^a(y) + \varphi^p(x) \otimes \tilde{\Omega}_3^p(y), \quad (4.45)$$

where the $\tilde{\Omega}_2^a$ are now harmonic two-forms on the CY base, as this decomposition could give rise to the required $U(1)$ gauge fields A^a in x-space. This time a runs over the $h^{(1,1)}$ possible two-cycles on the internal space, while p runs over the $2h^{(2,1)}$ possible three-cycles. We have also denoted the harmonic three-forms by $\tilde{\Omega}_3^q$.

Moduli space of M-theory compactifications

The M-theory description of the $E_8 \times E_8$ string that we have been using so far now leads to the following puzzle. To allow a decomposition of the three-form field of the kind that we want means that the background C_3 flux would have to be switched on parallel to the $M5_{||}$–brane. This is impossible for M-theory compactified on $S^1/\mathbb{Z}_2$ because the $\mathbb{Z}_2$ projection demands

$$C_3 \;\to\; -C_3, \quad (4.46)$$

and therefore all components of the background G-flux with no legs along the $S^1/\mathbb{Z}_2$ direction are projected out! Our naive compactification of M-theory on CY $\times S^1/\mathbb{Z}_2$ therefore cannot give rise to charged modes propagating on the string, making the situation at hand rather subtle.

However, in a cosmological setting an $E_8 \times E_8$ heterotic string in the limit of strong coupling cannot simply be described by a time-independent M-theory background. Instead the description should be in terms of a much bigger moduli space of M-theory compactifications, with the moduli themselves evolving with time. Specifically, we require a large moduli space of M-theory compactifications that would include the heterotic compactification above, at least for $t = 0$. Such a picture can be motivated from the well-known F-theory/heterotic duality which relates F-theory compactified on a K3 manifold to heterotic string theory compactified on a two-torus T^2 [65, 66, 67]. From here it follows immediately that M-theory compactified on K3 will be dual to heterotic string theory compactified on a three-torus T^3. Fibering both sides of the duality by another T^3 gives us

$$\begin{aligned} &\text{M theory on a } G_2 \text{ holonomy manifold } \equiv \\ &\text{Heterotic string theory on } \mathcal{M}_6, \end{aligned} \quad (4.47)$$

where the G_2 holonomy manifold is a seven-dimensional manifold given by a non-trivial T^3 fibration over a K3 base, and $\mathcal{M}_6$ is a six-dimensional manifold given by a non-trivial T^3 fibration over a T^3 base. Note that $\mathcal{M}_6$ is not in general a CY space. This duality has been discussed in the literature [68].

To confirm that there exists a point in the M-theory moduli space that describes the $E_8 \times E_8$ heterotic string, one needs to study the degeneration limits of the elliptically fibred base K3 (which can be written as a T^2 fibration over a P^1 base). Elliptically fibred K3 surfaces can be described by the family of elliptic curves (called Weierstrass equations)

$$y^2 \;=\; x^3 + f(z)x + g(z), \quad (4.48)$$

where (x, y) are the co-ordinates of the T^2 fibre of K3 and z is a co-ordinate on P^1, and f and g are polynomials of degree 8 and 12 respectively. Different moduli branches exist for which the modulus τ of the elliptic fibre is constant [69]. Gauge symmetries arise from the singularity types of the fibration on these branches. $E_8 \times E_8$ *can* be realised: The specific degeneration limit of K3 that produces an $E_8 \times E_8$ heterotic string corresponds to the Weierstrass equation [66, 69]:

$$y^2 = x^3 + (z - z_1)^5 (z - z_2)^5 (z - z_3)(z - z_4). \quad (4.49)$$

The two zeroes of order 5 each give rise to an E_8 factor, while the simple zeroes give no singularity.[11]

Given the existence of such a point in the moduli space of M-theory compactification, the future evolution of the system will in general take us to a different point in the moduli space. The picture that emerges from here is rather interesting. We start with heterotic $E_8 \times E_8$ theory. The strong coupling effects take us to the M-theory picture. From here cosmological evolution will drive us to a general point in the moduli space of G_2 manifolds. In fact, no matter where we start off, we will eventually be driven to some point in the vast moduli space of G_2 manifolds.

With M-theory compactified on a G_2 manifold, turning on fluxes becomes easy. However there are still a few subtleties that we need to address. Firstly, in the presence of fluxes we only expect the manifolds to have a G_2 *structure* and not necessarily G_2 holonomy.[12] Thus the moduli space becomes the moduli space of G_2 structure manifolds.[13] Secondly, due to Gauss' law constraint we will have to consider a non-compact seven manifold, much like the one considered in [72].[14] Finally, since our

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[11] This point in the moduli space of the M-theory compactification could as well be locally an $S^1/\mathbb{Z}_2$ fibration over a six-dimensional base $\widetilde{\mathcal{M}}_6$ (we haven't verified this here). Then the theory is dual to the $E_8 \times E_8$ heterotic string compactified on $\widetilde{\mathcal{M}}_6$, and there is a clear distinction between $M5_{||}$ and $M5_{\perp}$. Our earlier stability analysis could then be used to eliminate $M5_{\perp}$.

[12] For details on G_2 structure, see for example [70].

[13] As should be clear, we are no longer restricted to K3 fibered cases only. This situation is a bit like that of conifold transitions where we go from one CY moduli space to another in a cosmological setting governed by rolling moduli [71]. Furthermore, the constraint of G_2 structure comes from demanding low-energy supersymmetry. Otherwise we could consider any seven-manifold.

[14] Note that although the seven manifold is non-compact, the six-dimensional base is always compact here. Thus our earlier arguments depending on the existence of closed compact cycles on

M5–brane wraps a four-cycle inside the seven-manifold and we are switching on G fluxes parallel to the directions of the wrapped M5–brane, we need to address the concern of [73] that this is not permitted.

In the presence of a G-flux on the four-cycle a wrapped M5–brane has the following equation of motion:[15]

$$dH_3 = G. \tag{4.50}$$

For a four-cycle with no boundary this implies $G = 0$, as in [73]. However, our case is slightly different. We have a wrapped M5–brane on a four-cycle, but the G-flux has two legs along the wrapped cycle (the $x^{4,5}$ directions, say) and two legs in the $x^{0,1}$ directions. Therefore the G-flux is defined on a *non-compact* four-cycle and we can turn it on if we modify the above equation (4.50) by inserting n M2–branes ending on the wrapped M5–brane. The M2–branes end on the M5 in small loops of string in the $x^{4,5}$ directions, with their other ends at some point along the non-compact direction inside the seven-manifold, which the M2-branes are extended along. These strings will change (4.50) to

$$dH_3 = G - n\sum_{i=1}^{n} \delta^4_{\mathbf{W}^i}, \tag{4.51}$$

where the $\delta^4_{\mathbf{W}^i}$ denote the localised actions of n worldsheets on the M5–brane.[16] Then G need no longer be vanishing. In fact,

$$\int_{\widetilde{\Sigma}_4} G = n, \tag{4.52}$$

where $\widetilde{\Sigma}_4$ is the non-compact 4-cycle. This way we see that (a) we can avoid the $\mathbb{Z}_2$ projection (4.46) by going to a generic point in the moduli space of G_2 –structure manifolds, and (b) we can switch on a non-trivial G-flux along an M5–brane wrapped on a non-compact 4-cycle. Using the decompositions (4.43) and (4.45) we can now factorise the interaction term:

$$\begin{aligned} S_{int} &= -\frac{3}{2}\int (H_3 - C_3)\wedge\star(H_3 - C_3) + ... \\ &= -\frac{3}{2}\int (dB^+ - C_3)\wedge\star(dB^+ - C_3) + ... \\ &= -\frac{3}{2}\int \left(d\phi^a - A^a\right)\wedge\star\left(d\phi^b - A^b\right)\otimes\Omega_2^a\wedge\star\tilde{\Omega}_2^b \\ &\quad -\frac{3}{2}\int d^2x\sqrt{-h_x}\varphi^p\varphi^q\Omega_3^p\wedge\star\tilde{\Omega}_3^q + ... \end{aligned} \tag{4.53}$$

where the dotted terms above involve the n tadpoles coming from the worldvolume strings. These tadpoles would be proportional to ϕ^a. The variables h_x and h_y denote the determinants of the worldvolume metrics along the x and y directions respectively. We are interested in the coupling to electromagnetism, so we focus on the first term of (4.53) and take the number of 2-cycles on Σ_4 to be 1.[17] Then we have

$$S_{int} \;=\; -\frac{3}{2}\kappa\int d^2\sigma|d\phi - A|^2\sqrt{-h_x} + \ldots, \tag{4.54}$$

where

$$\kappa \;=\; \int_y \Omega_2\wedge\star\Omega_2 \tag{4.55}$$

is a constant factor.[18]

Fermionisation

The coupling in (4.54) implies that the action can be expressed more conveniently as one generating fermionic superconductivity along the string. We can see this by rewriting the term in terms of fermions, using a process known as fermionisation.

Fermionisation[19] is possible because of the equivalence in $1+1$ dimensions of the conformal field theories of $2n$ Majorana fermions and n bosons.[20]

The correlator for the bosonic field can be found from the action,[21]

$$S_B \;=\; \frac{1}{4\pi}\int d^2z\partial X^\mu(z,\bar{z})\bar{\partial}X^\nu(z,\bar{z}), \tag{4.56}$$

a CY_3 still hold, for an undetermined number of such cycles on some compact six-dimensional base. This is a construction we are free to choose.

[15] This can be seen from (4.40): one has to find the equation of motion for B^+ and then impose anti-selfduality of H_3.

[16] From the Type IIB point of view, this is analogous to the baryon vertex with spikes coming out from the wrapped D3–brane on a $\mathbf{S}^3$ with H_{RR} fluxes in the geometric transition set up [74].

[17] In the presence of multiple 2-cycles we will have more abelian fields. This doesn't change the physics of our discussion here.

[18] Note that there would also be non-abelian gauge fields coming from G fluxes *localised* at the singularities of the G_2 structure manifolds in the limit where some of the singularities are merging. The G flux that we have switched on is non-localised. This picture is somewhat similar to the story developed in [75] where heterotic gauge fields were generated from localised G fluxes on an eight manifold. In a time-dependent background all these fluxes would also evolve with time, but for our present case it will suffice to assume a slow evolution so that the gauge fields (abelian and non-abelian) do not fluctuate very fast.

[19] Canonical references are [77], [78] and [79]. [17] of [80] gives a comprehensive list of the early references. A useful textbook treatment is given in [76].

[20] This has been shown to hold in the infinite volume limit as well as in the finite volume case, where care must be taken to match the boundary conditions correctly [39]. Our long cosmic strings correspond to the infinite volume case.

[21] We use the conventions of Polchinski [76], working in units where $\alpha' = 2$.

to be

$$\langle X^\mu(z)X^\nu(w)\rangle \;=\; -\eta^{\mu\nu}\ln(z-w); \tag{4.57}$$
$$\langle X^\mu(z)\partial X^\nu(w)\rangle \;=\; \eta^{\mu\nu}\frac{1}{(z-w)}; \tag{4.58}$$
$$\langle \partial X^\mu(z)\partial X^\nu(w)\rangle \;=\; -\eta^{\mu\nu}\frac{1}{(z-w)^2}, \tag{4.59}$$

where z and w are local complex co-ordinates on the worldsheet and the correlators are all for the holomorphic (left-moving) parts of the bosonic fields only. The kinetic term for Majorana fermions on the worldsheet is

$$S_F \;=\; \frac{1}{4\pi}\int d^2z\left(\psi^\mu\bar{\partial}\psi_\mu + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu\right). \tag{4.60}$$

The fields ψ and $\tilde{\psi}$ are holomorphic and antiholomorphic respectively, with the holomorphic correlator given by

$$\langle\psi^\mu(z)\psi^\nu(w)\rangle \;=\; \eta^{\mu\nu}\frac{1}{(z-w)}. \tag{4.61}$$

Equivalently we could write the action and correlators in terms of

$$\psi \;=\; \frac{1}{\sqrt{2}}(\psi^1 + \imath\psi^2), \tag{4.62}$$
$$\bar{\psi} \;=\; \frac{1}{\sqrt{2}}(\psi^1 - \imath\psi^2),$$

as

$$S_F \;=\; \frac{1}{4\pi}\int d^2z(\bar{\psi}\bar{\partial}\psi + \psi\bar{\partial}\bar{\psi}) \tag{4.63}$$

(writing the holomorphic terms only). Then

$$\langle\psi(z)\bar{\psi}(w)\rangle \;=\; \frac{1}{(z-w)}.$$

These correlators lead one to make the identification

$$\psi(z) \;\equiv\; e^{\imath\phi(z)}; \tag{4.64}$$
$$\bar{\psi}(z) \;\equiv\; e^{-\imath\phi(z)},$$

where ϕ is the holomorphic part of one bosonic field. Now we consider the OPEs [76],

$$e^{\imath\phi(z)}e^{-\imath\phi(-z)} \;=\; \frac{1}{2z} + \imath\partial\phi(0) + 2zT_B^\phi(0) + \ldots \tag{4.65}$$
$$\psi(z)\bar{\psi}(-z) \;=\; \frac{1}{2z} + \psi\bar{\psi}(0) + 2zT_B^\psi(0) + \ldots,$$

where T_B^ϕ and T_B^ψ are the energy-momentum tensors arising from the actions (4.56) and (4.60):

$$T_B \;=\; -\frac{1}{2}\partial X^\mu\partial X_\mu - \frac{1}{2}\psi^\mu\partial\psi_\mu. \tag{4.66}$$

The identification (4.64) implies that the OPEs (4.65) should be equivalent, since all local operators in the two theories can be built from operator products of the fields being identified. This implies that the energy-momentum tensors of the two theories must be the same, allowing us to identify the theories as CFTs. This allows us to rewrite the kinetic term for n scalars as the kinetic term of a theory containing $2n$ fermions. Furthermore, we have the identification

$$\psi\bar{\psi} \;\equiv\; \imath\partial\phi. \tag{4.67}$$

We can now rewrite our wrapped M-brane term

$$\begin{aligned} |d\phi - A|^2 &= (\partial_\mu\phi - A_\mu)(\partial^\mu\phi - A^\mu) \\ &= \partial_\mu\phi\partial^\mu\phi - A_\mu\partial^\mu\phi - A^\mu\partial_\mu\phi + A^2 \end{aligned}$$

in a fermionic basis:[22]

$$\begin{aligned} |d\phi - A|^2 &= 2(\bar{\psi}\bar{\partial}\psi + \psi\bar{\partial}\bar{\psi}) + 2\imath A\psi\bar{\psi} + 2A\bar{A} \\ &= 2\psi_1(\bar{\partial} + \frac{\imath}{2}A)\psi_1 + 2\psi_2(\bar{\partial} + \frac{\imath}{2}A)\psi_2 \\ &\quad +2A\psi_1\psi_2 + 2A\bar{A}, \end{aligned} \tag{4.68}$$

which makes it clear that the worldsheet supports charged fermionic modes. Here A and $\bar{A}$ are defined in terms of components as in (4.62). Each boson is replaced by one ψ fermion and one $\bar{\psi}$ fermion at the same point, moving left at the speed of light, and carrying charge as shown explicitly by (4.68). This proves the existence of charged fermionic zero modes on the string obtained by suitably wrapping an M5-brane. Note that [32] gives a similar discussion, relating a theory describing charged fermionic zero modes trapped on a string to a bosonised dual with an interaction of the form $|d\phi - A|^2$.

One might worry that the above analysis should hold equivalently for the antiholomorphic part of the bosonic fields, leading to an equal number of right-moving fermionic modes. This is not the case, since ϕ is in fact holomorphic. From the anti-selfduality of dB^+ it follows that $d\phi = -\star d\phi$ in 1+1-dimensions.[23] Writing $d\phi$ as $(\partial + \bar{\partial})\phi$ one can show that $\bar{\partial}\phi = 0$ is implied by the anti-selfduality condition. This is just the condition that ϕ does not depend on $\bar{z}$, i.e. it is holomorphic or, in worldsheet terms, left-moving.

Axionic Stability

Finally, we should argue that the axionic instability is also removed for our case. This can be easily seen either directly from M-theory or from its type IIA limit. From a type IIA point of view the wrapped M5-brane

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[22] We make use of the fact that ϕ is holomorphic, as discussed below.

[23] This conclusion also depends on the fact that we have chosen a Calabi-Yau (or 6-d base of our 7-manifold) with only one 2-cycle on the 4-cycle Σ_4.

can appear as a D4-brane or an NS5-brane in ten dimensions depending on which direction we compactify in M-theory. First, assume that the four-cycle Σ_4 on which we have the wrapped M5-brane is locally of the form $\Sigma_3 \times S^1$. Then M-theory can be compactified along the S^1 direction to give a wrapped D4-brane on Σ_3 in ten dimensions[24]. We can now eliminate the axion following Becker, Becker and Krause [4]. The axion here appears from the D4-brane source i.e the five-form RR-charge C_5. This form descends to an axion in four dimensions exactly as we discussed before (dC_5 descends to dC_2 in four dimensions, which in turn is Hodge dual to $d\phi$, the axion). What are the gauge fields that will eat the axion? In [4] case the gauge fields arose on the ten-dimensional boundary. Here instead of the boundary, we can insert coincident D8 branes[25] that allow gauge fields to propagate on their world volume Σ_8. Therefore the relevant parts of the action are:

$$-\frac{1}{\kappa_{10}^2}\int |dC_5|^2 + \mu_8 \int_{\Sigma_8} C_5 \wedge \mathrm{tr}\ F \wedge F - \frac{1}{g_{\mathrm{YM}}^2}\int_{\Sigma_8} |F|^2 \tag{4.69}$$

which dimensionally reduce to an action similar to (4.33). This implies that the D8-brane gauge fields can eat up the axion to become heavy, and in turn eliminate the axionic instability. One subtlety with this process is the global D8-brane charge cancellation once we compactify. In fact a similar charge cancellation condition should also arise for the M2-branes that we introduced earlier to allow non-trivial fluxes on the M5 branes. We need to keep one of the internal direction non-compact to satisfy Gauss' law.[26]

If instead we dimensionally reduce in a direction orthogonal to the wrapped M5 brane, then one can show that it is impossible to eliminate the axionic instability by the above process. There might exist an alternative way to eliminate the axionic instability, but we haven't explored it here.

[24] One might worry at this stage that this is not the standard $\mathrm{M5}_{||}$ that we want. Recall however that at a generic point of the moduli space $\mathrm{M5}_{||}$ and $\mathrm{M5}_{\perp}$ cannot be distinguished.

[25] Such D8 branes are allowed in massive type IIA theory. They correspond to M9-branes when lifted to M-theory [81]. One can reduce an M9 as either a nine-brane in type IIA theory or a D8-brane. The nine-brane configuration is exactly dual to the $E_8 \times E_8$ theory that we discussed before, where the required O9-plane comes from Gauss' law constraint. To avoid the orientifold of the nine-brane configuration in type IIA, we consider only D8-branes in type IIA.

[26] A fully compactified version would require a much more elaborate framework that we do not address here.

Stability and superconductivity

At this point we pause to discuss the different types of cosmic strings permitted and the question of whether or not they can be superconducting. In general, cosmic strings can be either global (as in the case of Brandenberger and Zhang [2]) or local (as in the case of Becker, Becker and Krause [4])[45]. Superconductivity can also arise in two ways [32, 53]. Global strings can be superconducting thanks to an anomalous term of the form in (3.3) which causes charge to flow into the string, as explored by Kaplan and Manohar [26] (earlier references are [54] and [55]). For local strings (which are local with respect to the axion), this term is no longer gauge invariant.[27] Superconductivity is still possible if charged zero modes, either fermionic or bosonic, are supported on the gauge strings [32]. In that case a coupling of the form of (4.54) or (4.68) exists on the worldsheet. As we have seen, although the heterotic cosmic strings constructed by Becker, Becker and Krause [4] are local, they are not superconducting. A more general set-up is required in order for fermionic zero modes to be permitted, which is what we have constructed. Thus ours are local superconducting strings, where the superconductivity is clearest in a fermionic basis, as in (4.68).

D. Production of $\mathrm{M5}_{\|}$–branes

Whether strings or branes of a particular type will be present at late cosmological times relevant to the generation of seed galactic magnetic fields will depend on the history of the early universe. We must distinguish between cosmological models which underwent a phase of cosmic inflation (of sufficient length for inflation to solve the horizon problem of standard cosmology) and those which did not. Standard big bang cosmology, Pre-Big-Bang cosmology [56], Ekpyrotic cosmology [57] and string gas cosmology [58] are models in the latter class.

In models without inflation in which there was a very hot thermal stage in the very early universe all types of stable particles, strings and branes will be present. Hence, in such models one expects all stable branes to be present. Since the wrapped $\mathrm{M2}_{\perp}$–branes are stable but have too large a tension for the values of the parameters considered here, we conclude that there is a potential problem for our proposed magnetogenesis scenario without a period of inflation which would eliminate the $\mathrm{M2}_{\perp}$–branes present in the hot early universe. However, if the temperature was never hot enough to thermally produce the $\mathrm{M2}_{\perp}$–branes, as may well happen in string gas cosmology or in bouncing cosmologies, there would be no cosmological $\mathrm{M2}_{\perp}$–brane problem.[28]

[27] Thanks to Louis Leblond for pointing this out to us.

[28] Another way to get rid of the potential $\mathrm{M2}_{\perp}$–brane problem

On the other hand, in inflationary universe scenarios, the number densities of all particles, strings and branes present before the period of inflation was red-shifted. To have any strings or branes present after inflation within our Hubble patch, these objects must be generated at the end of the period of inflation. Which objects are generated will depend critically on the details of the inflationary model. Since we are focusing on a M-theory realisation of a particular heterotic string compactification, we will first discuss the issue of generation of cosmic superstrings in the context of a concrete realization of inflation in heterotic string theory due to Becker, Becker and Krause [59]. In this model, several M5–branes are distributed along the $\frac{S^1}{\mathbb{Z}_2}$ interval. During the inflationary phase these are sent towards the boundaries by repulsive interactions. Slow-roll conditions are satisfied as long as the distance d between the M5 branes is much less than L the orbifold length. Once $d \sim L$ non-perturbative contributions which stabilise the orbifold length and Calabi-Yau volume at values consistent with a realistic value for G_N and a SUSY-breaking scale close to a TeV come into effect. This stabilisation was used in the argument above and also leads to a small $\mathrm{M5}_{\parallel}$ tension, so that while wrapped M5–branes will be produced at the end of inflation there is insufficient energy density to produce the $\mathrm{M2}_{\perp}$–branes.

In our model, where cosmological evolution takes us to a generic point in the moduli space of G_2 structure manifolds (by rolling moduli), there may not be a problem with $\mathrm{M2}_{\perp}$–branes $-$ at least in the limit of compact G_2 structure manifolds with G_2 holonomy. This is because compact manifolds with G_2 holonomy have finite fundamental group. This implies vanishing of the first Betti number [82], which in turn means that M2–branes have no 1-cycles to wrap on. Once we make the G_2 manifolds non-compact (keeping the six-dimensional base compact with vanishing first Chern class[29]) we can still argue the non existence of finite 1-cycles, and therefore we don't expect a cosmological M2–brane problem.

E. Amplitude of the induced seed magnetic fields

Finally, we estimate the magnitude of the resulting seed magnetic fields, making use of the same arguments used in [2]. We want to calculate the magnetic field at a time t after decoupling in the matter-dominated epoch (specifically, at the beginning of the period of galaxy formation) at a distance r from the string. We will take this distance to be a typical galactic scale.

The magnetic field strength is given by (3.6). The coefficients c_+ and c_- can be determined as in [26] by solving the anomalous Maxwell equations (3.4) at the radius of the string core r_c given a string current with

$$\lambda = \frac{en}{2\pi}, \tag{4.70}$$

where n is the number per unit length of charge carriers on the string, all of which are moving relativistically. The result is [2, 26]

$$B(r) \sim \frac{en}{2\pi}\left(\frac{r}{r_c}\right)^{\alpha\pi} r^{-1}. \tag{4.71}$$

During the formation of the string network at time t_c, the number density of charge carriers is of the order of T_c (where $T(t)$ is the temperature at the time t):

$$n(t_c) \sim T_c. \tag{4.72}$$

As the correlation length $\xi(t)$ of the string network expands, the number density drops proportionally to the inverse correlation length. However, mergers of string loops onto the long strings leads to a buildup of charge on the long strings which can be modelled as a random walk [2] and partially cancels the dilution due to the expansion of the universe.[30] Taken together, this yields

$$n(t) \sim \left[\frac{\xi(t_c)}{\xi(t)}\right]^{1/2} n(t_c). \tag{4.73}$$

Assuming that the universe is dominated by radiation between t_c and t_{eq} and by matter from t_{eq} until t, we can express the ratio of correlation lengths in terms of ratios of temperatures (using $a(t) \sim T^{-1}$), with the result

$$n(t) \sim \left[\frac{T(t)}{T_{eq}}\right]^{3/4} \frac{T_{eq}}{T(t)} n(t_c). \tag{4.74}$$

Upon insertion of (4.73) and (4.72) into (4.71) one finds

$$B(t) \sim \frac{e}{2\pi}\frac{T_{eq}}{r}\left[\frac{T(t)}{T_{eq}}\right]^{3/4}\left(\frac{r}{r_c}\right)^{\alpha\pi}. \tag{4.75}$$

By expressing the temperature in units of GeV and the radius in unit of 1m, and converting from natural units to physical units making use of the relation

$$\frac{e}{2\pi}\frac{GeV}{m} = 10^5 \text{ Gauss}, \tag{4.76}$$

we obtain

$$B(t) \sim 10^5 \text{ Gauss}\frac{T_{eq}}{\text{GeV}} r_M^{-1}\left[\frac{T(t)}{T_{eq}}\right]^{3/4} (\frac{r}{r_c})^{\alpha\pi}, \tag{4.77}$$

might be to change the parameters of the model in order to reduce the $\mathrm{M2}_{\perp}$–brane tension to an acceptable level.

[29] The base doesn't have to be a Calabi-Yau manifold to have vanishing first Chern class. See for example constructions in [75].

[30] Note that without string interactions, the correlation length $\xi(t)$ would not scale as t.

where r_m is the radius in units of meters.

Evaluated at the time of recombination t_{rec} (shortly after the time t_{eq} and at a radius of 1pc, the physical length which turns into the current galaxy radius after expansion from t_{rec} to the current time, we obtain

$$B(t) \sim 10^{-20} \text{ Gauss} \left(\frac{r}{r_c}\right)^{\alpha\pi} . \quad (4.78)$$

Even with $\alpha = 0$ (our case), the value is of the same order of magnitude as is required to yield the seed magnetic field for an efficient galactic dynamo. If there were an anomalous coupling of our string to electromagnetism, the amplitude would be greatly enhanced since r_c is a microscopic scale whereas r is cosmological.

V. DISCUSSION AND CONCLUSIONS

We have proposed a mechanism to generate seed magnetic fields which are coherent on galactic scales based on a M-theory realisation of a particular heterotic string compactification. According to our proposal, wrapped M5–branes, which generically settle to a point in the moduli space of G_2 structure manifolds, act as superconducting cosmic strings from the point of view of our four-dimensional universe. These branes are stable, and carry charged zero modes which are excited via the Kibble mechanism in the early universe. Because of the scaling properties of cosmic string networks, the currents on the strings resulting from the charged zero modes generate magnetic fields which are coherent on the scale of the cosmic string network. This scale is proportional to the Hubble distance at late times, which means that the scale increases much faster in time than the physical length associated with a fixed comoving scale. It is this scaling which enables our mechanism to generate magnetic fields that are coherent on galactic scales at the time of galaxy formation.

Our set-up is a possible string theoretic realisation of the proposal made by Brandenberger and Zhang in [2]. The mechanism of [2] was based on pion strings which are unstable after the time of recombination [31], while the strings in our mechanism are stable. Thus, our current scenario predicts the existence of seed fields which are coherent on all cosmological scales, in contrast to the mechanism of [2] which admits a maximal coherence scale. This means our mechanism is in principle distinguishable from that of [2]. However, it is only seed fields on scales which undergo gravitational collapse which can be amplified by the galactic dynamo mechanism. The fields which we predict on larger scales will not have been amplified and thus will have a very small amplitude. These weak coherent fields are therefore a prediction of our set-up, but their amplitude is presumably beyond our current detection abilities.

We have studied the viability of all branes arising in M-theory as sources of the superconducting cosmic strings required for our magnetic field generation mechanism. At a special point in the moduli space of G_2 structure manifolds where locally we have M-theory on $\frac{S^1}{\mathbb{Z}_2}$ fibered over a six-dimensional base, we can use tension and stability analyses to rule out all but the M5$_{||}$–brane, as summarised in the table below (see [4] for details):

	topology	tension	stability	production
$M2_\perp$	✓	×	✓	×
$M2_{\|\|}$	×	-	-	-
$M5_\perp$	✓	✓	×	×
$M5_{\|\|}$	✓	✓	✓	✓

The wrapped M5$_{||}$–brane in the $E_8 \times E_8$ heterotic theory realisation compactified to 3+1 dimensions avoids the instability pointed out by Witten [3]. Under cosmological evolution by rolling moduli, our system is driven to a generic point in the moduli space of G_2 structure manifolds where we also expect non-trivial G fluxes evolving with time. At this point, under some reasonable assumptions, M2$_\perp$–branes cannot exist (no finite 1-cycles) and there is not much difference between M5$_\perp$ and M5$_{||}$ branes. Thus for this M5–brane to be a valid candidate for producing primordial seed magnetic fields via the mechanism proposed in [2], we needed to verify that the brane can carry a superconducting current generated via charged zero modes at any generic point in the moduli space of G_2 structure manifolds. We have shown that this is indeed true. Thus the wrapped M5–brane could supply the desired seed magnetic fields directly from string (or M) theory.

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APPENDIX A: THE GALACTIC DYNAMO

In this appendix we give a brief overview of the galactic dynamo mechanism [7, 11].

The interstellar medium is turbulent because of stellar winds, supernova explosions and hydro-magnetic instabilities. This turbulence is rendered cyclonic by the non-uniform rotation of the gaseous disc of the galaxy, which means that it gains a net helicity (while individual eddies can possess helicity, the mean helicity in a non-rotating body averages out to zero). These two effects, cyclonic turbulence and non-uniform rotation, are the key ingredients of what is called the $\alpha\omega$ dynamo, shown

by Parker [10, 14] to be responsible for the magnetic field of the galaxy. The dynamo mechanism also provides an explanation for the specific field configurations observed in spiral galaxies [1]. It is now thus the accepted explanation [31] for regeneration and amplification of the magnetic field in spiral galaxies (elliptical galaxies and clusters are non-rotating or slowly rotating, and coherent large-scale fields are not observed in them, an observation which provides further support for the dynamo explanation. Only small-scale local dynamos can operate in these systems [9]).

The dynamo mechanism can be explained heuristically as follows: any poloidal field (in the meridional plane, which lies perpendicular to the plane of the galactic disc) will generate field lines in the azimuthal direction thanks to the non-uniform rotation. At the same time, cyclonic motion produces poloidal field from azimuthal field. This process is shown schematically in Figure 2, where a cyclonic cell is shown raising and twisting the azimuthal field B_ϕ into a loop with non-vanishing projection in the meridional plane. Such loops are produced on scales comparable to the size of the largest turbulent eddies and then mixed and smoothed by general turbulence until they coalesce into a general poloidal field. The twist that makes the convective cell cyclonic is supplied by the Coriolis force [10].

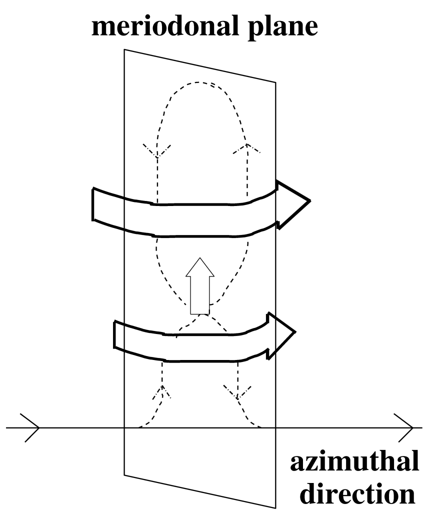

FIG. 2: A cyclonic convective cell distorts and twists a magnetic field line in the azimuthal direction (solid black) into the meridional plane, generating a poloidal field line (dashed). Taken from [10].

[31] Criticisms of the model and its assumptions are reviewed by Kulsrud [17]; the author concludes that although some issues merit closer examination, none are serious enough to cast doubt on the dynamo as the most likely generator of galactic fields.

Formally, solution of the dynamo equations in a slab of gas, representing the galactic disc, produces regenerative modes in the azimuthal direction, for boundary conditions allowing magnetic flux to escape from the slab. Diffusion within the slab and diffusive escape from the surface of the slab are both essential to the operation of the dynamo because they permit the escape of reversed fields which would otherwise cause active degeneration [12, 14, 17].

These effects can be seen from the hydro-magnetic equation

$$\frac{\partial \vec{B}}{\partial t} \;=\; \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)+\eta\vec{\nabla}^2\vec{B}-\vec{\nabla}\eta\times\left(\vec{\nabla}\times\vec{B}\right), \tag{A.1}$$

which governs the large-scale behaviour of magnetic fields. $\vec{v}$ is the laminar velocity and $\eta\sim\frac{1}{\sigma}$ the turbulent resistivity. $\vec{\nabla}^2\vec{B}$ is the dissipative term, and $\vec{\nabla}\times(\vec{v}\times\vec{B})$ the inductive term. Note that the loop in the meridional plane sketched in Figure 2 will produce an emf, written in general as

$$\epsilon_i \;=\; \alpha_{ij}B_j+\eta_{ijk}\frac{\partial B_j}{\partial x_k}, \tag{A.2}$$

which will enter the induction equation as

$$\vec{\nabla}\times(\vec{v}\times\vec{B}) \;\rightarrow \vec{\nabla}\times(\vec{v}\times\vec{B}+\vec{\epsilon}). \tag{A.3}$$

The first term corresponds to the helical part of the turbulence (labelled by α) and η_T is the turbulent diffusion coefficient. The dynamo equations then follow from (A.1) and are given by [13]

$$\begin{aligned}\left[\frac{\partial}{\partial t}-\eta\left(\nabla^2-\frac{1}{r^2}\right)\right]A_\phi &= \Gamma B_\phi; \qquad (A.4)\\ \left[\frac{\partial}{\partial t}-\eta\left(\nabla^2-\frac{1}{r^2}\right)\right]B_\phi &= B_r\left(\frac{\partial V_\phi}{\partial r}-\frac{V_\phi}{r}\right),\end{aligned}$$

where Γ is a measure of the mean rate and strength of the cyclonic motions and V_ϕ is the rotational velocity. The $\alpha\omega$ dynamo can operate in any differentially rotating body, and is accepted as the primary mechanism for the maintenance of magnetic fields in the sun and the galaxy [9].

[1] R. Beck, A. Brandenburg, D. Moss, A. Shukurov and D. Sokoloff, "Galactic Magnetism: Recent developments and perspectives," Ann. Rev. Astron. Astrophys. **34**, 155 (1996).

[2] R. H. Brandenberger and X. M. Zhang, "Anomalous global strings and primordial magnetic fields," Phys. Rev. D **59**, 081301 (1999) [arXiv:hep-ph/9808306].

[3] E. Witten, "Cosmic Superstrings," Phys. Lett. B **153**,

243 (1985).

[4] K. Becker, M. Becker and A. Krause, "Heterotic cosmic strings," Phys. Rev. D **74**, 045023 (2006) [arXiv:hep-th/0510066].

[5] E. J. Copeland, R. C. Myers and J. Polchinski, "Cosmic F- and D-strings," JHEP **0406**, 013 (2004) [arXiv:hep-th/0312067].

[6] E. Fermi, "On the Origin of the Cosmic Radiation," Phys. Rev. **75**, 1169 (1949).

[7] E. N. Parker, "Cosmical Magnetic Fields: Their Origin and Their Activity," Clarendon Press, Oxford (1979).

[8] M. S. Turner and L. M. Widrow, "Inflation Produced, Large Scale Magnetic Fields," Phys. Rev. D **37**, 2743 (1988).

[9] L. M. Widrow, "Origin of Galactic and Extragalactic Magnetic Fields," Rev. Mod. Phys. **74**, 775 (2003) [arXiv:astro-ph/0207240].

[10] E. N. Parker, "The Generation of Magnetic Fields in Astrophysical Bodies III. Turbulent Diffusion of Fields and Efficient Dynamos," Astrophys. J. **163**, 279 (1971).

[11] H. K. Moffat, "Magnetic Field Generation in Electrically Conducting Fluids," CUP, Cambridge (1978).

[12] E. N. Parker, "Hydromagnetic Dynamo Models," Astrophys. J. **122**, 293 (1955).

[13] E. N. Parker, "The Generation of Magnetic Fields in Astrophysical Bodies I. The Dynamo Equations," Astrophys. J. **162**, 665 (1970).

[14] E. N. Parker, "The Generation of Magnetic Fields in Astrophysical Bodies II. The Galactic Field," Astrophys. J. **163**, 255 (1971).

[15] M. J. Rees, "The Origin and Cosmogonic Implications of Seed Magnetic Fields," Q. Jl. Roy. Astron. Soc. **28**, 197 (1987).

[16] L. Mestel and K. Subramanian, "Galactic Dynamos and Density Wave Theory," Mon. Not. Roy. Astron. Soc. **248**, 677 (1991).

[17] R. M. Kulsrud, "A Critical Review Of Galactic Dynamos," Ann. Rev. Astron. Astrophys. **37**, 37 (1999).

[18] A. Vilenkin and E. P. S. Shellard, "Cosmic Strings and Other Topological Defects," Cambridge University Press (1994).

[19] M. B. Hindmarsh and T. W. B. Kibble, "Cosmic strings," Rept. Prog. Phys. **58**, 477 (1995) [arXiv:hep-ph/9411342].

[20] R. H. Brandenberger, "Topological defects and structure formation," Int. J. Mod. Phys. A **9**, 2117 (1994) [arXiv:astro-ph/9310041].

[21] T. W. B. Kibble, "Topology Of Cosmic Domains And Strings," J. Phys. A **9**, 1387 (1976);
T. W. B. Kibble, "Phase Transitions In The Early Universe," Acta Phys. Polon. B **13**, 723 (1982).

[22] A. Albrecht and N. Turok, "Evolution Of Cosmic Strings," Phys. Rev. Lett. **54**, 1868 (1985);
A. Albrecht and N. Turok, "Evolution Of Cosmic String Networks," Phys. Rev. D **40**, 973 (1989).

[23] D. P. Bennett and F. R. Bouchet, "Evidence For A Scaling Solution In Cosmic String Evolution," Phys. Rev. Lett. **60**, 257 (1988);
D. P. Bennett and F. R. Bouchet, "High Resolution Simulations of Cosmic Sring Evolution. 1. Network Evolution," Phys. Rev. D **41**, 2408 (1990).

[24] B. Allen and E. P. S. Shellard, "Cosmic String Evolution: A Numerical Simulation," Phys. Rev. Lett. **64**, 119 (1990).

[25] X. Zhang, T. Huang and R. H. Brandenberger, "Pion and eta strings," Phys. Rev. D **58**, 027702 (1998) [arXiv:hep-ph/9711452].

[26] D. B. Kaplan and A. Manohar, "Anomalous Vortices and Electromagnetism," Nucl. Phys. B **302**, 280 (1988).

[27] J. P. Ostriker, A. C. Thompson and E. Witten, "Cosmological Effects of Superconducting Strings," Phys. Lett. B **180**, 231 (1986).

[28] T. Vachaspati, "Magnetic fields from cosmological phase transitions," Phys. Lett. B **265**, 258 (1991).

[29] R. H. Brandenberger, A. C. Davis, A. M. Matheson and M. Trodden, "Superconducting cosmic strings and primordial magnetic fields," Phys. Lett. B **293**, 287 (1992) [arXiv:hep-ph/9206232].

[30] K. Dimopoulos and A. C. Davis, "Cosmological consequences of superconducting string networks," Phys. Lett. B **446**, 238 (1999) [arXiv:hep-ph/9901250];
D. Battefeld, T. Battefeld, D. H. Wesley and M. Wyman, "Magnetogenesis from Cosmic String Loops," JCAP **0802**, 001 (2008) [arXiv:0708.2901 [astro-ph]];
T. Battefeld, "Magnetogenesis from Rotating Cosmic String Loops," AIP Conf. Proc. **957**, 305 (2007) [arXiv:0709.0735 [astro-ph]].

[31] M. Nagasawa and R. H. Brandenberger, "Stabilization of embedded defects by plasma effects," Phys. Lett. B **467**, 205 (1999) [arXiv:hep-ph/9904261];
M. Nagasawa and R. Brandenberger, "Stabilization of the electroweak Z string in the early universe," Phys. Rev. D **67**, 043504 (2003) [arXiv:hep-ph/0207246].

[32] E. Witten, "Superconducting Strings," Nucl. Phys. B **249**, 557 (1985).

[33] N. Turok and R. H. Brandenberger, "Cosmic Strings And The Formation Of Galaxies And Clusters Of Galaxies," Phys. Rev. D **33**, 2175 (1986).

[34] A. Stebbins, "Cosmic Strings and Cold Matter," Ap. J. (Lett.) **303**, L21 (1986).

[35] H. Sato, "Galaxy Formation by Cosmic Strings," Prog. Theor. Physics **75**, 1342 (1986).

[36] J. Silk and A. Vilenkin, "Cosmic Strings And Galaxy Formation," Phys. Rev. Lett. **53**, 1700 (1984);
T. Vachaspati, "Cosmic Strings and the Large-Scale Structure of the Universe," Phys. Rev. Lett. **57**, 1655 (1986).

[37] A. Stebbins, S. Veeraraghavan, R. H. Brandenberger, J. Silk and N. Turok, "Cosmic String Wakes," Astrophys. J. **322**, 1 (1987).

[38] M. Wyman, L. Pogosian and I. Wasserman, "Bounds on cosmic strings from WMAP and SDSS," Phys. Rev. D **72**, 023513 (2005) [Erratum-ibid. D **73**, 089905 (2006)] [arXiv:astro-ph/0503364];
A. A. Fraisse, "Constraints on topological defects energy density from first year WMAP results," arXiv:astro-ph/0503402;
N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, "Fitting CMB data with cosmic strings and inflation," Phys. Rev. Lett. **100**, 021301 (2008) [arXiv:astro-ph/0702223].

[39] M. Green, J. Schwarz and E. Witten, "Superstring Theory, Vols. 1 & 2" (Cambridge Univ. Press, Cambridge, 1987).

[40] A. Vilenkin and A. E. Everett, "Cosmic Strings And Domain Walls In Models With Goldstone And Pseudogoldstone Bosons," Phys. Rev. Lett. **48**, 1867 (1982).

[41] V. Braun, Y. H. He, B. A. Ovrut and T. Pantev, "The exact MSSM spectrum from string theory," JHEP **0605**,

043 (2006) [arXiv:hep-th/0512177].
[42] R. Blumenhagen, S. Moster and T. Weigand, "Heterotic GUT and standard model vacua from simply connected Calabi-Yau manifolds," Nucl. Phys. B **751**, 186 (2006) [arXiv:hep-th/0603015].
[43] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, "A mini-landscape of exact MSSM spectra in heterotic orbifolds," Phys. Lett. B **645**, 88 (2007) [arXiv:hep-th/0611095].
[44] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, "The Heterotic Road to the MSSM with R parity," Phys. Rev. D **77**, 046013 (2008) [arXiv:0708.2691 [hep-th]].
[45] J. Polchinski, "Open heterotic strings," JHEP **0609**, 082 (2006) [arXiv:hep-th/0510033].
[46] L. Leblond and S. H. H. Tye, "Stability of D1-strings inside a D3-brane," JHEP **0403**, 055 (2004) [arXiv:hep-th/0402072].
[47] J. Polchinski, "Introduction to cosmic F- and D-strings," [arXiv:hep-th/0412244].
[48] P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. B **475**, 94 (1996) [arXiv:hep-th/9603142].
[49] R. A. Battye and J. Weller, "Cosmic structure formation in hybrid inflation models," Phys. Rev. D **61**, 043501 (2000) [arXiv:astro-ph/9810203].
[50] F. A. Jenet *et al.*, "Upper bounds on the low-frequency stochastic gravitational wave background from pulsar timing observations: Current limits and future prospects," Astrophys. J. **653**, 1571 (2006) [arXiv:astro-ph/0609013].
[51] G. Curio and A. Krause, "Four-flux and warped heterotic M-theory compactifications," Nucl. Phys. B **602**, 172 (2001) [arXiv:hep-th/0012152],
[52] G. Curio and A. Krause, "Enlarging the parameter space of heterotic M-theory flux compactifications to phenomenological viability," Nucl. Phys. B **693**, 195 (2004) [arXiv:hep-th/0308202],
A. Krause, "Heterotic M-theory, warped geometry and the cosmological constant problem," Fortsch. Phys. **49**, 163 (2001).
[53] D. D. Harari and F. D. Mazzitelli, "Electromagnetic radiation from a global cosmic string in an external magnetic field," Phys. Lett. B **266**, 269 (1991).
[54] G. Lazarides and Q. Shafi, "Superconducting Strings In Axion Models," Phys. Lett. B **151**, 123 (1985).
[55] C. G. . Callan and J. A. Harvey, "Anomalies And Fermion Zero Modes On Strings And Domain Walls," Nucl. Phys. B **250**, 427 (1985).
[56] M. Gasperini and G. Veneziano, "Pre - big bang in string cosmology," Astropart. Phys. **1**, 317 (1993) [arXiv:hep-th/9211021].
[57] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, "The ekpyrotic universe: Colliding branes and the origin of the hot big bang," Phys. Rev. D **64**, 123522 (2001) [arXiv:hep-th/0103239].
[58] A. Nayeri, R. H. Brandenberger and C. Vafa, "Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology," Phys. Rev. Lett. **97**, 021302 (2006) [arXiv:hep-th/0511140];
R. H. Brandenberger and C. Vafa, "Superstrings in the Early Universe," Nucl. Phys. B **316**, 391 (1989).
[59] K. Becker, M. Becker and A. Krause, "M-theory inflation from multi M5-brane dynamics," Nucl. Phys. B **715**, 349 (2005) [arXiv:hep-th/0501130].
[60] K. Subramanian, "Can the turbulent galactic dynamo generate large-scale magnetic fields?," [arXiv:astro-ph/9707280].
[61] G. W. Gibbons and P. K. Townsend, "Vacuum interpolation in supergravity via super p-branes," Phys. Rev. Lett. **71**, 3754 (1993) [arXiv:hep-th/9307049].
[62] D. M. Kaplan and J. Michelson, "Zero Modes for the D=11 Membrane and Five-Brane," Phys. Rev. D **53**, 3474 (1996) [arXiv:hep-th/9510053].
[63] K. Dasgupta and S. Mukhi, "Orbifolds of M-theory," Nucl. Phys. B **465**, 399 (1996) [arXiv:hep-th/9512196];
E. Witten, "Five-branes and M-theory on an orbifold," Nucl. Phys. B **463**, 383 (1996) [arXiv:hep-th/9512219].
[64] P. K. Townsend, "D-branes from M-branes," Phys. Lett. B **373**, 68 (1996) [arXiv:hep-th/9512062].
[65] C. Vafa, "Evidence for F-Theory," Nucl. Phys. B **469**, 403 (1996) [arXiv:hep-th/9602022].
[66] D. R. Morrison and C. Vafa, "Compactifications of F-Theory on Calabi–Yau Threefolds – I," Nucl. Phys. B **473**, 74 (1996) [arXiv:hep-th/9602114].
[67] D. R. Morrison and C. Vafa, "Compactifications of F-Theory on Calabi–Yau Threefolds – II," Nucl. Phys. B **476**, 437 (1996) [arXiv:hep-th/9603161].
[68] M. Atiyah and E. Witten, "M-theory dynamics on a manifold of G(2) holonomy," Adv. Theor. Math. Phys. **6**, 1 (2003) [arXiv:hep-th/0107177];
B. Acharya and E. Witten, "Chiral fermions from manifolds of G(2) holonomy," [arXiv:hep-th/0109152];
E. Witten, "Anomaly cancellation on G(2) manifolds," [arXiv:hep-th/0108165].
[69] K. Dasgupta and S. Mukhi, "F-theory at constant coupling," Phys. Lett. B **385**, 125 (1996) [arXiv:hep-th/9606044].
[70] S. Chiossi and S. Salamon, "The intrinsic torsion of $SU(3)$ and G_2 structures," Proc. of International Conference on Differential Geometry, [arXiv:math/0202282].
[71] P. Candelas, P. S. Green and T. Hubsch, "Rolling Among Calabi-Yau Vacua," Nucl. Phys. B **330**, 49 (1990);
A. Strominger, "Massless black holes and conifolds in string theory," Nucl. Phys. B **451**, 96 (1995) [arXiv:hep-th/9504090].
[72] K. Becker and M. Becker, "Compactifying M-theory to four dimensions," JHEP **0011**, 029 (2000) [arXiv:hep-th/0010282].
[73] M. J. Duff, R. Minasian and E. Witten, "Evidence for Heterotic/Heterotic Duality," Nucl. Phys. B **465**, 413 (1996) [arXiv:hep-th/9601036].
[74] F. Cachazo, K. A. Intriligator and C. Vafa, "A large N duality via a geometric transition," Nucl. Phys. B **603**, 3 (2001) [arXiv:hep-th/0103067];
M. Becker, K. Dasgupta, A. Knauf and R. Tatar, "Geometric transitions, flops and non-Kaehler manifolds. I," Nucl. Phys. B **702**, 207 (2004) [arXiv:hep-th/0403288];
S. Alexander, K. Becker, M. Becker, K. Dasgupta, A. Knauf and R. Tatar, "In the realm of the geometric transitions," Nucl. Phys. B **704**, 231 (2005) [arXiv:hep-th/0408192];
M. Becker, K. Dasgupta, S. H. Katz, A. Knauf and R. Tatar, "Geometric transitions, flops and non-Kaehler manifolds. II," Nucl. Phys. B **738**, 124 (2006) [arXiv:hep-th/0511099];
R. Gwyn and A. Knauf, "The Geometric Transition Re-

visited," [arXiv:hep-th/0703289].

[75] K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088];
K. Becker, M. Becker, K. Dasgupta and P. S. Green, "Compactifications of heterotic theory on non-Kaehler complex manifolds. I," JHEP **0304**, 007 (2003) [arXiv:hep-th/0301161];
K. Becker, M. Becker, P. S. Green, K. Dasgupta and E. Sharpe, "Compactifications of heterotic strings on non-Kaehler complex manifolds. II," Nucl. Phys. B **678**, 19 (2004) [arXiv:hep-th/0310058].

[76] J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond," *Cambridge, UK: Univ. Pr. (1998) 531 p*

[77] S. R. Coleman, "Quantum sine-Gordon equation as the massive Thirring model," Phys. Rev. D **11**, 2088 (1975).

[78] S. Mandelstam, "Soliton operators for the quantized sine-Gordon equation," Phys. Rev. D **11**, 3026 (1975).

[79] E. Witten, "Nonabelian bosonization in two dimensions," Commun. Math. Phys. **92**, 455 (1984).

[80] W. Siegel and B. Zwiebach, "Gauge String Fields," Nucl. Phys. B **263**, 105 (1986).

[81] E. Bergshoeff, M. de Roo, M. B. Green, G. Papadopoulos and P. K. Townsend, "Duality of Type II 7-branes and 8-branes," Nucl. Phys. B **470**, 113 (1996) [arXiv:hep-th/9601150]; E. Bergshoeff and J. P. van der Schaar, "On M-9-branes," Class. Quant. Grav. **16**, 23 (1999) [arXiv:hep-th/9806069].

[82] D. D. Joyce, "Compact Manifolds of Special Holonomy," Oxford University Press, 2000.